UCRL- 95340
PREPRINT

RHEOLOGICAL BEHAVIORS OF AN ELASTOMERIC MEMBRANE

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This paper prepared for presentation at 11th Canadian Congress of Applied Mechanics, May 31 - June 4, 1987, Edmonton, Canada

September 1986

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The inflation of a plane circular membrane was first studied experimentally by Treloar[1] and analytically by Adkins and Rivlin[2]. The same problem with different constitutive equations was solved by Klingbeil and Shield[3]. Later, Yang and Feng[4] further extended the formulation to a class of axisymmetric membranes of Mooney material under large deformations. The membranes studied have usually been elastomeric materials, which often exhibit viscoelastic behavior. To date, very few problems pertaining to nonlinear viscoelastic membranes have been investigated because of the inherent complexities associated with nonlinearity both in geometry and in material. In this paper, we demonstrated that viscoelastic solutions can be obtained.

The formulation centers on the determination of the deformed configuration of a plane circular membrane under an inflating pressure. Two sets of polar cylindrical coordinates are used. The coordinates $(R,\Psi,0)$ are used to describe the undeformed midsurface of the membrane. The coordinates $[r(R),\Psi,z(R)]$ are used to describe the deformed midsurface. The stretch ratios λ_1 and λ_2 in the meridian and circumferential directions, respectively, are

$$\lambda_1 = \left[(r')^2 + (z')^2\right]^{1/2}, \qquad \lambda_2 = r/R \tag{1}$$

In the above and subsequent equations, the primes denote the differentiation with respect to R, and all quantities are evaluated at time t_n unless otherwise specified. The time t_n is at the nth increment of Δt. The value for Δt is a small quantity but it need not be a constant. Another geometric quantity, θ is the angle between a vector normal to the midsurface of a deformed membrane and the axis of symmetry. The relationship between the angle and the stretch ratios is

$$\theta = \cos^{-1}\left[(R\lambda_2' + \lambda_2)/\lambda_1\right] \tag{2}$$

The nonlinear viscoelastic constitutive equation used here was developed by Christensen[5]. For incompressible materials and the assumption that for a thin membrane the stress in the normal direction is small, the constitutive equation for the normal Cauchy stress in the meridian direction reduces to

$$\sigma_1 = g_0\left(\lambda_1^2 - \frac{1}{\lambda_1^2\lambda_2^2}\right) + \frac{\lambda_1^2}{2}\int_0^{t_n} g_1(t_n - T)\,\frac{\partial\lambda_1^2(T)}{\partial T}\,dT - \frac{1}{2\lambda_1^2\lambda_2^2}\int_0^{t_n} g_1(t_n - T)\frac{\partial}{\partial T}\left(\frac{1}{\lambda_1^2(T)\lambda_2^2(T)}\right)dT \tag{3}$$

and a similar equation for σ_2, the normal Cauchy stress in the circumferential direction. There are two material property functions, g_0 and g_1. The g_0 term is the long-term elastic material constant and the $g_1(t)$ is a viscoelastic relaxation function. For sufficiently slow processes, the integral terms in the constitutive equation become negligibly small and the remaining terms constitute the theory of rubber elasticity. Therefore, the constitutive equation can be used for studying both elastic and viscoelastic effects.

The coordinate values that describe the deformed configuration are unknown functions of the undeformed coordinates as well as the inflating pressure (P). These unknown functions can be determined from the force equilibrium equations in the tangential and normal directions. The equilibrium equations are reduced to a set of nonlinear, differential-integral equations.

In order to simplify the numerical procedures for solving these equations, two steps are taken. First, the governing equations are written in terms of three new variables: λ_1, λ_2, and θ. Second, a recurrence formula, developed by Feng[6], is used. With the recurrence formula, it is not necessary to recalculate the hereditary integrals in the convolution operation at each new value of time, and as a result, the governing equations are reduced to three first-order ordinary differential equations for each time step. When combined with the compatibility equation derived from equation (2), the governing equations can be cast in a standard form

$$\begin{aligned} \lambda_1' &= F_1\left[\lambda_1,\lambda_2,\theta,\lambda_1(t_{n-1}),\lambda_2(t_{n-1}),\lambda_1'(t_{n-1}),R\right] \\ \lambda_2' &= F_2(\lambda_1,\lambda_2,\theta,R) \\ \theta' &= F_3\left[\lambda_1,\lambda_2,\theta,\lambda_1(t_{n-1}),\lambda_2(t_{n-1}),R,P\right] \end{aligned} \tag{4}$$

* Work performed under the auspices of the U. S. Department of Energy by the Lawrence Livermore National Laboratory under Contract W-7405-ENG-48.

With specified boundary and initial conditions, solutions to equations (4) can be obtained by numerical integration.

The numerical results for creep and relaxation of a plane circular membrane under an inflating pressure are obtained. The relaxation function, written in terms of the exponential law, is used here:

$$g_1(t) = c \exp(-t/\tau) \tag{5}$$

where c is a material constant and τ is a relaxation time constant.

In order to show the viscoelastic effect of deformation on an elastomeric membrane, we performed an experiment. Before designing the apparatus for the test, however, we needed to decide whether to do a creep or a relaxation test. From the theoretical study, one can obtain a relationship between the inflating pressure and the deformation. It shows clearly that there is an instability point at which the initial rise of the inflating pressure will be followed by a fall as the deformation increases. Therefore, if one performs the creep test, there are two or more membrane configurations for each inflating pressure. Furthermore, the pressure-deformation curve is relatively flat beyond the instability point; this will produce very inaccurate results. For these reasons, we decided to do a relaxation test instead of a creep test.

In the experiment, the membrane is made of latex rubber and clamped between two plates. The thickness of the membrane is 0.0762 cm. The radius of the membrane is 2.54 cm. The membrane is inflated by air from a reservoir. The height of the deformed membrane at the pole is fixed at 3 cm. which corresponds to the stretch ratio at the pole of 2.0. An infrared photosensor controls the maximum height of the deformed membrane at the pole. In order to decrease the pressure during the relaxation test, a "leaky" valve is provided. The photosensor is connected to a solenoid valve that closes the air intake valve when the photosensor light path is blocked by the membrane, and opens the air intake valve when the membrane deflates. The infrared photosensor keeps the height of the deformed membrane at the pole to within 0.001 cm., and the pressure transducer is sensitive to within five pascals. The pressure transducer, coupled with a digital voltage converter, is used for data acquisition. Data is automatically recorded by a microcomputer. The experimental results, as a graph of inflation pressure versus time, demonstrate the viscoelastic effect of latex rubber.

In this paper, the nonlinear viscoelastic membrane mechanics have been formulated, solved, and experimentally verified for the inflation of an initially flat thin circular membrane. With some minor modifications on these formulations, the whole class of axisymmetric viscoelastic membrane problems can be studied. In the formulation, the geometric nonlinearity as well as the material nonlinearity are included. The theoretical value for the strains can be as large as needed; for practical applications, however, these values are limited by the fracture strength and are usually less than a few hundred percent. In the formulation, the relaxation function is represented by a single exponential term that does not represent the material properties in reality. However, the formulation presented here can be extended to a more realistic relaxation function which is represented by a series of exponential functions or by the power law. The analytical results combined with the experimental results can be used further to determine the material properties of elastomers, g_0 and $g_1(t)$.

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